Subject SP5

Corrections to 2023 study material

0 Comment

This document contains details of any errors and ambiguities in the Subject SP5 study materials for the 2023 exams that have been brought to our attention. We will incorporate these changes in the study material each year. We are always happy to receive feedback from students, particularly details concerning any errors, contradictions or unclear statements in the courses. If you have any such comments on this course please email them to SP5@bpp.com.

This document was last updated on **5 January 2023**.

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1 Course Notes

Background on LIBOR (non examinable)

In 2012, it was discovered that some banks were fraudulently mis-reporting the interest rates which they were submitting (and colluding with other banks in doing so) in order to manipulate LIBOR.

In July 2017, Andrew Bailey, then Chief Executive of the Financial Conduct Authority (FCA), indicated that LIBOR's role was unsustainable.

From January 2022, publication of most LIBOR settings ended – the exception being for contracts still in force where publication will continue until 2023.

The 2023 Core Reading states that:

In the UK, money market interest rates were historically often quoted relative to LIBOR (the London Inter Bank Offered Rate). LIBOR has been replaced by a series of reference rates such as the Sterling Overnight Index Average (SONIA) or for US Dollars the Secured Overnight Funding Rate (SOFR).

Background to the corrections (non examinable)

Some updates made to the Core Reading and Course Notes from 2022 to 2023, simply replaced references to LIBOR with references to new alternatives such as SONIA and/or SOFR. However, these alternative reference floating interest rates do not have the same technical characteristics as LIBOR, and so cannot be regarded as simple substitutes.

For example, LIBOR rates are *forward-looking* interest rates determined by asking a panel of banks to estimate rates at which they could borrow from other banks over a variety of terms. SONIA/SOFR are *backward-looking* overnight interest rates based on actual transactions. Also, whilst LIBOR incorporated some credit risk, SONIA and SOFR are considered to be (very nearly) risk-free.

The following corrections aim to address this issue by generalising what were past references to LIBOR to become references to 'forward looking floating rates', rather than references to SONIA and/or SOFR.

Chapter 4, Section 3.1, 'Example'

The first paragraph should end '… based on the 1-year *forward-looking* (spot) reference rate.'

Chapter 4, Section 3.1, Question and Solution

The heading in the second column of the tables should be '*Forward-looking* reference rate % at start of year'.

Chapter 4, Section 3.4, 'Forwards'

In the final paragraph of ActEd text on page 27, the text in brackets now reads: '(as the first floating rate payment will be based on the level of the *forward-looking* floating rate at outset)'.

Chapter 12, Section 1.8 'Interest rate swaps'

The references here should be to using discount rates which are forward-looking floating rates. Substitute pages have been provided towards the end of this update document.

Chapter 12, Section 1.9 'Valuation of a swap as a series of forward rate agreements'

The reference rate in the example should not be SOFR, but rather an (un-named, appropriate) forward-looking floating rate. Substitute pages have been provided towards the end of this update document.

Chapter 13, Section 1.4 'Example'

In the example, 'LIBOR' should be corrected to read 'the reference rate'.

Chapter 13, Section 1.4 'Pricing interest rate caps'

In the self-assessment question and its solution, the references should be to a forward-looking floating rate (not SONIA). Substitute pages have been provided towards the end of this update document.

Chapter 13, Section 1.8, 'Interest rate collars'

The references in the example should be to a forward-looking floating rate (not SONIA). Substitute pages have been provided towards the end of this update document.

Chapter 23, Section 1.4, 'Reducing the cost of borrowing'

The references in the interest rate swap example should be to a forward-looking floating rate (not SONIA). Substitute pages have been provided towards the end of this update document.

Chapter 23, Practice questions - question 23.6

All references to 'SONIA' should be corrected to read 'reference rate'.

Further reading (non examinable)

If you want to read more about the issues raised by the demise of LIBOR, then the following document is perhaps a good place to start: https://www.bankofengland.co.uk/-/media/boe/files/markets/benchmarks/what-you-need-to-know-about-libor-transition.

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2 X Assignments

Assignment X1 – Question X1.5

There is an error in the solution to SP5 X1.5 (iii). The present value of the euro payments should be €118.656*m* and so the net present value of the swap should be 118.656 – 115.643 = €3.013*m.*

The full solution to this part should read as follows:

The present value in Euro of the payments received by Company X can be valued as a 3-year euro bond:

$$B_€ = (0.055 \times 120)e^{-0.0575} + (0.055 \times 120)e^{-0.0575\times 2} + (1.055 \times 120)e^{-0.0575\times 3}$$ [1]

This gives a value of payments received of €118.656*m*. [½]

The present value in Sterling of the Sterling payments payable by Company X can be valued as a 3-year Sterling bond:

$$B_£ = (0.045 \times 100)e^{-0.05} + (0.045 \times 100)e^{-0.05\times 2} + (1.045 \times 100)e^{-0.05\times 3}$$ [1]

This gives a value of payments payable of £98.296*m*. [½]

The current euro value of the sterling bond is therefore:

$$\frac{98.296}{0.85} = €115.643m$$ [1]

Therefore, the net present value of the swap to Company X is equal to:

118.656 – 115.643 = €3.013*m* [1]

[Total 5]

Assignment X3 – Question X3.3

Question X3.3 should be adjusted to refer throughout to 'forward-looking reference rates' (not SOFR).

In contrast an investor with a long forward position will not be affected in this way by interest rate movements. Thus, all else being equal, a long futures contract will be less attractive than the equivalent long forward contract. Futures prices will therefore tend to be lower than (un-margined) forward prices.

So, (un-margined) forward prices will be higher than the corresponding futures prices. Consequently, forward rates must be lower than the corresponding futures rates, as is illustrated by the answers to parts (ii) and (iii).

1.8 Interest rate swaps

As we saw in Chapter 4 an interest rate swap can be valued as a long position in one bond compared to a short position in another bond. The cashflows are usually discounted using a reference rate like (a forward looking version of) **SONIA or SOFR zero-coupon interest rates (since this reference rate is the cost of funds for a financial institution).**

This involves an implicit assumption that the risk associated with the swap cashflows is the same as that associated with loans in the interbank market.

Thus:

$$V_{swap} = B_{fl} - B_{fix}$$

where:

- B_{fix} **is the value of the fixed-rate bond underlying the swap**
- B_{fl} **is the value of the floating-rate bond underlying the swap.**

Here we are valuing the swap to the party that has lent money at the variable interest rate (and so is receiving variable interest rate payments under the swap) and is borrowing money at the fixed rate.

The fixed-rate bond is valued in the usual way:

$$B_{fix} = \sum_{i=1}^{n} k e^{-r_i t_i} + L e^{-r_n t_n}$$

where:

- **the cashflows are k at time t_i $(1 \le i \le n)$ and L at time t_n**
- r_i is the continuously-compounded reference zero rate for maturity t_i.

Value of floating-rate bond immediately after a payment date

The value of the floating-rate bond will be L immediately after a payment date.

Here we are implicitly assuming that the floating-rate bond pays a forward-looking market rate of interest and the same variable rates are used both to estimate the future coupon payments and to discount them. So, unlike with fixed interest bonds, there is no discrepancy between the market price and the redemption value.

The same idea applies to an interest-only variable-rate mortgage, where the outstanding amount of the loan on Day 1 (the value) is of course equal to the amount of the loan (the principal) regardless of the pattern of interest rates on that day.

We can illustrate this general result by means of the following simple example.

Example

Consider a 3-year floating-rate bond with annual coupons and a principal of 100. The coupon paid at the end of each year is based on the value of the floating interest rate at the start of that year.

Let:

- R_k $(k=1,2,3)$ be the k-year zero rate
- R_{Fk} $(k=1,2,3)$ be the forward rate applicable over year k.

If both series of rates are assumed to be annually compounded then the price of the bond can be written as:

$$P=100\left[\frac{R_{F1}}{1+R_1}+\frac{R_{F2}}{(1+R_2)^2}+\frac{1+R_{F3}}{(1+R_3)^3}\right]$$

Recall that:

- $R_{F1}=R_1$
- $\left(1+R_2\right)^2=\left(1+R_1\right)\left(1+R_{F2}\right)$
- $\left(1+R_3\right)^3=\left(1+R_1\right)\left(1+R_{F2}\right)\left(1+R_{F3}\right)$

Using these results to substitute in for R_{F1}, R_{F2} and R_{F3} in the bond price expression gives:

$$P=100\left[\frac{R_1}{1+R_1}+\frac{1}{1+R_1}-\frac{1}{(1+R_2)^2}+\frac{1}{(1+R_1)(1+R_{F2})}\right]$$

Here the final two terms cancel. So the whole expression simplifies to 100 – *ie* the principal amount. This result can be generalised to allow for any term and frequency of coupon payments.

Value of floating-rate bond immediately before a payment date

The same argument also applies *immediately after* any coupon payment during the lifetime of a floating-rate bond (as the remaining part of the original bond can be thought of as a new floating rate bond with a shorter term). It can therefore be used to value such a bond immediately *before* a payment date.

Immediately before a payment date, its value will be $L+k^*$, where k^* is the floating-rate payment that will be made on the next payment date due at time t_1. That is, equal to the sum of the value immediately after the payment plus the amount of the payment itself.

Thus the value of the bond today is its value just before the next payment date, discounted at rate r_1 for time t_1:

$$\boldsymbol{B_{fl} = (L + k^*)e^{-r_1 t_1}}$$

Question

Consider an interest rate swap in which Company X has agreed to:

- receive 6% *pa* fixed
- pay 1-year forward-looking floating reference rate

both on a notional principal of $50 million.

The outstanding term of the swap is currently 2½ years, and the current forward-looking zero rates for ½, 1½ and 2½ years are 5.7%, 5.75% and 5.85% *pa* respectively, all compounded continuously. Interest payments are made annually, with the next due in 6-months' time.

If the 1-year forward-looking reference rate was 5.72% *pa* (compounded annually) six months ago, calculate the value of the swap to Company X.

Solution

The value of the swap to Company X, which is *receiving* 6% *pa* fixed in return for paying 1-year floating reference rate is given by:

$$V_{swap} = B_{fix} - B_{fl}$$

The general expression for the value of the fixed bond is:

$$B_{fix} = \sum_{i=1}^{n} ke^{-r_i t_i} + Le^{-r_n t_n}$$

which in this case is equal to:

$$B_{fix} = 3e^{-0.057\times 0.5} + 3e^{-0.0575\times 1.5} + 53e^{-0.0585\times 2.5} = 51.4567$$

The general expression for the value of the floating-rate bond is:

$$B_{fl} = (L + k^*)e^{-r_1 t_1}$$

which in this case is equal to:

$$B_{fl} = (50 + 50\times 0.0572)e^{-0.057\times 0.5} = 51.3748$$

Thus:

$$V_{swap} = 51.4567 - 51.3748 = +\$0.0820m$$

1.9 Valuation of a swap as a series of forward rate agreements

Alternatively, the swap can be valued as a series of forward rate agreements.

The procedure is:

1. **Calculate forward rates for each of the reference rates that will determine swap cashflows.** This is done using the formula described in Section 1.4.

2. **Calculate swap cashflows on the assumption that the reference rates will equal the forward rates.** In other words, we assume that the forward rates will actually be realised.

3. **Set the swap value equal to the present value of these cashflows.** Here the cashflows are discounted using the appropriate reference zero rates.

Example

Consider the swap in the previous question.

In 6 months' time, the company will receive a fixed interest payment of 6% *pa* and pay a floating-rate payment based on the 1-year reference rate at the last payment date of 5.72% *pa*, both payments based on the principal of $50 million. The net present value of the cashflows then payable (discounting using the 6-month zero rate) is thus equal to:

$$50\times(0.06-0.0572)e^{-0.057\times½}=0.13607$$

In order to calculate the net present value of the cashflows in 18 months' time, we need to determine the amount of the floating-rate payment at that time. This first requires that we calculate the forward rate applicable over the 1-year period starting in 6 months' time.

The continuously-compounded 1-year forward rate for the period starting in six months' time is found using the usual formula:

$$R_F = \frac{R_2T_2 - R_1T_1}{T_2 - T_1}$$

Here we have:

$$R_F = \frac{0.0575\times1½-0.057\times½}{1½-½}=0.05775$$

The annually compounded forward rate R is thus given by:

$$1+R=e^{R_F}=e^{0.05775}$$

which gives the value of $R=0.05945$, *ie* 5.945% *pa*.

The floating-rate payment in 18 months' time is 50 × 0.05945 and the net present value of the cashflows exchanged at that time is:

$$50\times(0.06-0.05945)e^{-0.0575\times1½}=0.02523$$

Similarly, to value the cashflows payable in 2½ years' time we first need to find the forward rate for the year ending at that time. This turns out to be 6% *pa* compounded continuously, or 6.1837% *pa* compounded annually. The net present value of the interest payments exchanged in 2½ years' time is therefore:

$$50\times\left(0.06-0.061837\right)e^{-0.0585\times 2½}=-0.07933$$

We do not need to consider the principal here as it is not exchanged in practice.

The total value of the swap is therefore:

$$0.13607 + 0.02523 - 0.07933 = \$0.082 \text{ million}$$

as before.

Question

Show that the annually-compounded 1-year forward rate for the period starting in 18 months' time is equal to 6.1837% *pa.*

Solution

The continuously-compounded 1-year forward rate for the period starting in 18 months' time is found using the usual formula:

$$R_F = \frac{R_2T_2 - R_1T_1}{T_2 - T_1}$$

Here we have:

$$R_F = \frac{0.0585\times 2½-0.0575\times 1½}{2½-1½}=0.0600$$

The annually compounded forward rate R is thus given by:

$$1+R=e^{R_F}=e^{0.06}$$

which gives the required value of $R=0.061837$, *ie* 6.1837% *pa*.

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2 Arbitrage pricing and the concept of hedging

2.1 Introduction

Any two assets (or asset portfolios) that provide identical payoffs in all future times and conditions *must* have the same price. If this is not the case, then there are opportunities for *arbitrage* – by selling the more expensive portfolio and buying the cheaper portfolio with the proceeds, an investor could produce an unlimited return without capital expenditure.

As investors attempt to take advantage of this position, the demand for the cheaper portfolio will increase (causing its price to rise) and demand for the more expensive portfolio will fall. Equilibrium will be restored when the prices of the two portfolios are equal (note that we are making many assumptions here about tax, transaction costs, access to borrowing, divisibility of assets, investors' knowledge, *etc*).

The pricing of most derivatives is based on the assumption that markets are arbitrage-free, which is often a reasonable, if not exact, assumption in practice.

Arbitrage pricing of forward contracts

We can apply the no arbitrage concept to derive the price, F_0, of a forward contract in terms of the spot price S_0. No arbitrage requires that:

$$F_0 = S_0 e^{rT}$$

where T is the time when the forward contract matures and r is the risk-free rate of interest (for an investment maturing at time T). If this equality did not hold, arbitrage possibilities would exist.

Question

Complete the next section of notes yourself, by filling in the missing words and mathematical expressions for the amounts involved. You may wish to use a pencil in case you need to change your answer. (The passage relates to an asset with no associated cashflows – *eg* a non-dividend-paying share.)

If $F_0 < S_0 e^{rT}$ the investor can _____ the asset short at the current spot price ___, invest the sale proceeds risk-free (to accumulate a sum _______), and, at the same time, enter into a _____ forward contract to buy the asset at time T at price ___. This will generate a risk-free profit of __________ for no initial outlay, at time T.

Similarly, if $F_0 > S_0 e^{rT}$ unlimited profit can be made from a strategy of _________ S_0 now to buy the asset and entering into a ______ forward contract to sell the asset at time T for ___. At that time the loan and accumulated interest of ______ will be repayable, leaving the investor with a risk-free profit of _________.

The only price for the forward, F_0, that eliminates the arbitrage opportunities is _______.

where:

- σ **is the forward price volatility;**
- σ_y **is the corresponding forward yield volatility;**
- y_0 **is the initial forward yield on this bond;**
- ***D*** **is the (modified) duration of the forward bond underlying the option.**

The modified duration is given by:

$$D = \frac{\text{Duration}}{(1 + y/m)}$$

where *m* is the frequency per annum with which *y* is compounded.

Question

A 6-year zero-coupon bond is priced at 74.62%.

(i) What is the duration of the bond?

(ii) Calculate the annual effective yield on the bond and hence its modified duration.

Solution

(i) *Duration of the bond*

Given that the duration is the weighted average of the time to the payments provided by a bond, the duration of a zero-coupon bond must be equal to its outstanding term. Hence, in this case it is simply 6 years.

(ii) *Annual effective yield and modified duration*

The annual effective yield on the bond is found from:

$$74.62 = \frac{100}{(1+s_6)^6}$$

From which, $s_6 = 0.05$, *ie* 5% *pa.*

So the modified duration of the zero-coupon bond is equal to:

$$D = \frac{6}{1.05} = 5.71 \text{ years}$$

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1.4 Interest rate caps

How interest rate caps work

A popular over-the-counter interest rate option is the *interest rate cap*, which is designed to provide insurance against the rate of interest on an underlying floating rate note rising above a certain level. This level is known as the *cap rate*, R_X .

If you have a mortgage, you may well have a similar arrangement where there is a clause that 'caps' your interest payments if the mortgage rate goes above a specified level. So an interest rate cap is most likely to be of use if you wish to cap the variable interest rate you are *paying.*

Example

Consider a floating rate note, the interest rate on which is reset to equal the reference rate at pre-specified time intervals, say every 3 months. The interest rate on the floating rate note for the first 3 months is equal to initial reference rate on the starting date. The interest rate for the next three months is set equal to the reference rate that applies at the first 3-month point and so on.

Consider a 3-year interest rate cap based on a principal amount of \$1,000,000, in which the interest rate is reset every 3 months, and with an interest rate cap of 6% *pa* (compounded quarterly, as the payments are made quarterly).

If the relevant reference rate at the start of a particular 3-month period was equal to 7% *pa* (compounded quarterly), then the interest payment on a floating rate note would be:

$$0.25 \times 0.07 \times 1{,}000{,}000 = \$17{,}500$$

If the reference rate was equal to the cap interest rate of 6% *pa*, the interest payment would be:

$$0.25 \times 0.06 \times 1{,}000{,}000 = \$15{,}000$$

Given that the actual reference rate exceeds the cap rate, the interest rate cap will make a payment equal to the difference between the two payments, *ie* a payment of \$2,500. If instead, the actual reference rate had been less than the cap rate, then no payment would be made.

In practice:

- the payment, if any, occurs at the end of the 3-month period concerned (not at the start) to coincide with the interest payment due under the floating rate note, based on the interest rates prevailing at the start of the period

- usually no payment is made at the first reset date, even if the initial reference rate exceeds the cap rate, as the initial reference rate (and hence any payment) is known.

Thus, in this example, there would be 11 reset dates (at times 0.25, 0.50, …, 2.75) and 11 possible payment dates (at times 0.50, 0.75, …, 3.00).

Pricing interest rate caps

Suppose the interest rate R_K on the floating rate note is reset every three months to equal a reference interest rate (the time between resets is known as the *tenor*). The payoff provided by the cap will be:

$$L \times 0.25 \times \max(R_K - R_X, 0)$$

where:

- **L is the principal amount specified for the contract**
- R_X is the cap rate.

This payoff calculation will occur every three months during the life of the cap, *T*. Note that payment of the payoff occurs at the *end* of the tenor period (here three months) not at the time of calculation (the beginning of the period).

Thus the cap leads to a payoff at time t_{k+1} (k = 1, 2, . . . , n) of:

$$L\,\delta_k \max(R_K - R_X, 0)$$

where t_{n+1} is defined as T and the reset dates are $t_1, t_2, ..., t_n$.

Note that:

- $\delta_k = t_{k+1} - t_k$
- **R_K and R_X are expressed with a compounding frequency equal to the frequency of resets**
- there is no reset at t_0 and hence no payment at t_1.

Question

Suppose that Bank Z has sold an interest rate cap based on a principal of £50*m* and a fixed cap rate of 6% *pa* convertible quarterly.

Calculate the payments Bank Z must make to the purchaser of the cap at the end of each of the first six quarters if the forward-looking reference rate at the start of first quarter is 6.20%, and moves to 6.03%, 6.61%, 5.99%, 6.12% and 6.20% *pa* in subsequent quarters.

Solution

Recall that the cap leads to a payoff at the end of each quarter (except the first) equal to:

$$L\,\delta_k \max(R_K - R_X, 0)$$

where:

- $L = £50m$ is the principal
- $\delta_K = 1/4$ is the tenor

- R_K is the forward-looking reference rate at the start of the quarter
- $R_X = 6\%$ *pa* is the cap rate.

Thus, the payments made by Bank Z at the end of each quarter are:

Quarter	*1*	*2*	*3*	*4*	*5*	*6*
Forward-looking reference rate	6.20%	6.03%	6.61%	5.99%	6.12%	6.20%
$\max(R_K - R_X, 0)$	0.2%	0.03%	0.61%	0%	0.12%	0.20%
Payment	*	£3.75*k*	£76.25*k*	£0	£15*k*	£25*k*

** Note in the first quarter, there is no payoff as the initial reference rate is known, so any payoff would be known and simply priced in to the cap – hence it is usually ignored.*

The above expression suggests that we can think of each of the potential payments under an interest rate cap as the payoff from a call option on the reference rate.

Each payoff is a call option on the reference interest rate observed at time t_k (with the payoff occurring at time t_{k+1}) and is known as a *caplet.* The cap is a portfolio of *n* such options.

Recall Equation (2) in Section 1.1, which gives the general expression for the price of a European call option as:

$$c = P(0, T^*)[F_0\Phi(d_1) - X\Phi(d_2)]$$

If the rate R_K is assumed to be lognormally distributed with volatility σ_K, then we can use the expression in Section 1.1 above, substituting:

- **R_X for *X***
- **F_K (the forward rate for the period between time t_k and t_{k+1}) for F_0**
- **t_{k+1} for T^***
- **σ_K for σ**

to value each caplet (each caplet must be valued separately).

Once we allow for the fact that the actual cash payment will reflect both the principal *L* and the tenor δ_k, the resulting formula is:

$$c = L\delta_k P(0, t_{k+1})[F_K\Phi(d_1) - R_X\Phi(d_2)]$$

where:

- $d_1 = \left(\dfrac{\ln\left(F_K / R_X\right) + \sigma_k^2 t_k / 2}{\sigma_k \sqrt{t_k}} \right)$

- $d_2 = \left(\dfrac{\ln\left(F_K / R_X\right) - \sigma_k^2 t_k / 2}{\sigma_k \sqrt{t_k}} \right)$

- F_K is the forward rate between t_k and t_{k+1}.

The discount factor here is $P\left(0, t_{k+1}\right)$ because the payoff occurs at time t_{k+1}, based on the interest rate observed at time t_k.

Question

Suppose that the zero curve is flat at 7% *pa* for all terms and that the interest rate volatility is 10% *pa.* Consider a 3-year interest rate cap, based on annual payments, a principal of \$100*k* and a cap rate of 6% *pa.* Given that all interest rates are quoted as annual effective rates, calculate the value of the interest rate caplet in the third year.

Solution

In this instance:

- $L = 100k$
- $\delta_k = 1$
- $P\left(0, t_{k+1}\right) = \dfrac{1}{1.07^3} = 0.81630$
- $F_K = 0.07$
- $R_X = 0.06$
- $\sigma_k = 0.10$
- $t_k = 2$

So:

- $d_1 = \left(\dfrac{\ln\left(F_K / R_X\right) + \sigma_k^2 t_k / 2}{\sigma_k \sqrt{t_k}} \right) = \left(\dfrac{\ln\left(0.07/0.06\right) + 0.10^2 \times 2/2}{0.10\sqrt{2}} \right) = 1.1607$

- $d_2 = \left(\dfrac{\ln\left(F_K / R_X\right) - \sigma_k^2 t_k / 2}{\sigma_k \sqrt{t_k}} \right) = \left(\dfrac{\ln\left(0.07/0.06\right) - 0.10^2 \times 2/2}{0.10\sqrt{2}} \right) = 1.0193$

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So, the value of the caplet is:

$$c = L\delta_k P(0, t_{k+1})[F_K \Phi(d_1) - R_X \Phi(d_2)]$$
$$= 100k \times 0.81630[0.07 \times \Phi(1.1607) - 0.06 \times \Phi(1.0193)]$$
$$= \$869$$

1.5 Spot rate volatilities

As each caplet must be valued separately, **one issue to be considered is whether to use a different 'spot' volatility for each caplet or to assume the same 'flat' volatility for all the caplets comprising any particular cap (but to vary this volatility according to the life of the cap).**

When a different forward rate volatility is used to value each caplet, based on the term of that caplet, then the volatilities used are referred to as *spot volatilities.* If instead the same (average) volatility is used for all caplets within a given cap, then it is known as a *flat volatility.*

Spot and flat volatilities usually vary with the term of the option. In practice, a plot of spot volatilities against term usually produces a humped curve. The flat volatility curve is likewise humped, although typically less so, as flat volatilities are essentially cumulative averages of spot volatilities.

1.6 Interest-rate floors

An *interest rate floor* contract provides a payoff when the interest rate on an underlying floating rate note falls below a certain rate. It is otherwise identical to a cap. A floor can therefore provide insurance against a fall in a floating rate, which might be useful if you are *receiving* the floating rate.

Pricing interest rate floors

An equivalent approach (to that used to price interest rate caps) **is used for an *interest rate floor* contract providing payoff when the interest rate on an underlying floating-rate note falls below a certain rate. In this case, we are valuing a portfolio of put options**

Question

State a formula for the payoff provided by a floorlet at time t_{k+1} $(k = 1, 2, ..., n)$.

Solution

A floorlet provides a payoff if the actual reference rate R_K is less than the floor rate R_X. So the payoff provided by a floorlet at time t_{k+1} $(k = 1, 2, ..., n)$ is:

$$L\,\delta_k \max(R_X - R_K, 0)$$

Interest rate swap example

Assume that:

- BP can borrow at 50 bps (basis points) over Government bonds or 50 bps over 6-month floating reference rates.
- AB Foods can borrow at 30 basis points over Government bonds. However, because it is a cyclical company investors worry about its exposure to higher interest rates. Therefore if it borrows on a floating rate basis it has to pay 80 bps over 6-month floating reference rates.

BP therefore enjoys a comparative advantage at borrowing at the floating interest rate. This is because it pays the same 50 bp margin over 6-month SONIA as it does over the fixed rate, whereas AB Foods has to pay an additional 50 bp margin when borrowing at a floating rate when compared to borrowing at a fixed rate. Thus, BP can undertaking floating rate borrowing *relatively* more cheaply than AB Foods is able to.

Likewise, AB Foods enjoys a comparative advantage when borrowing at the fixed rate.

Note that the absolute numbers do not matter. Reducing borrowing costs via a swap is based entirely on relative and not absolute borrowing costs, *ie* comparative as opposed to absolute advantage.

Suppose that it is agreed that AB Foods pays the intermediary bank 6-month reference rate + 5 bps, and that BP pays it a fixed payment of the government bond yield minus 5 bps, then the following position can be achieved:

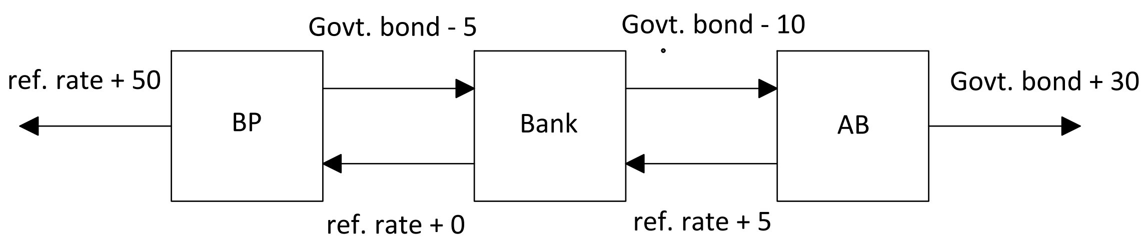

Figure 23.1: Interest rate swap between BP and AB Foods

BP is then, in effect, a net borrower of fixed interest at:

$$+ (\text{reference rate} + 50) - (\text{reference rate} + 0) + (\text{Government} - 5) = \text{Government} + 45$$

ie at the Government bond rate + 45 basis points. This is better than the Government bonds + 50 at which it could borrow by itself. In addition, AB Foods is, in effect, borrowing at reference rate + 45, rather than reference rate + 80.

Question

Show that AB Foods is borrowing at reference rate + 45 bps.

Solution

Under the swap, AB Foods ends up a net borrower at:

+ (Government + 30) – (Government – 10) + (reference rate + 5) = reference rate + 45

ie at variable rate + 45 basis points.

This only one of a number of possible solutions. Equally possible is:

- BP borrows at Government bonds + 30 (saving 20 basis points)
- AB Foods borrows at 6-month reference rate + 50 (saving 30 basis points).

In fact, there is a continuum of possible solutions, each involving a total saving of 50 basis points, which is split between the two companies, and the intermediary bank, in such a way that each is better off and so wishes to undertake the swap.

A similar situation may exist if different companies enjoy a comparative advantage when borrowing in different *currencies*. If each borrows in the currency in which they enjoy the comparative advantage, they can then use a currency rate swap to reduce the total cost of financing and both benefit from a lower cost of debt.

1.5 Currency

An investor which holds investments in a currency other than that in which its liabilities are denominated is exposed to variations in the exchange rate as well as variability in the return achieved on the underlying investment.

If the investor does not want to bear this exposure it can be hedged in various ways, for example by using forward currency contracts.

1.6 Inflation swaps

An investor which holds an asset that has an income stream that is linked to an inflation index is exposed to variations in future expectations of the level of inflation, and for longer-dated inflation-linked payments this can be a source of significant market risk.

An inflation swap allows a receiver of inflation-linked payments to pay these to a counterparty in return for receiving a fixed payment. Typical payers of inflation under inflation swaps will include holders of loans with inflation-linked payments or leaseholders who receive inflation-linked rental income. Institutional investors such as pension funds, with inflation-linked liabilities, can use inflation swaps to receive inflation and thereby hedge the market risk from uncertain future inflation within their liabilities.

For some time now, pension schemes have been reducing their equity weightings and increasing their holdings of bonds and liability matching assets. A means of gaining exposure to index-linked bonds is by means of an inflation swap. They have been achieving this in two ways:

1. real rate swaps
2. synthetic index-linked bonds.